Research Statement

1 Introduction

My research involves understanding algebraic and geometric structures, especially those found quantum field theories (QFTs). Throughout my PhD I have focused particularly on the $3d$ mirror symmetry of topologically twisted supersymmetric QFTs. In addition to this, I am also interested in understanding other duality phenomena such as holography and the geometric Langlands program. Mathematically, this has involved subjects including infinite-dimensional and non-semisimple representation theory; deformation quantization; vertex, KLR and factorization algebras; and symplectic singularities.

In Sections 2 I provide a brief introduction to $3d$ mirror symmetry and some background regarding boundary VOAs and line operators. In Section 3 I provide overviews of two papers:

- Section 3.1 summarises [SW24], a project with Ben Webster using cylindrical KLRW algebras to compute a tilting generator for $T^*\mathrm{Gr}(2,4)$ which is the resolved Coulomb branch of a quiver gauge theory.
- Section 3.2 summarises [FS24], a project with Andrea Ferrari proving that the boundary VOA of the A-twist of super quantum electrodynamics (SQED) is the simple quotient of the $\mathfrak{psl}(N|N)$ affine vertex superalgebra and that the associated variety of this VOA is the minimal nilpotent orbit of $\mathfrak{sl}(N)$ which is the Higgs branch of SQED.

In Section 4 I outline two projects I am currently working on:

- Section 4.1: in joint work with Andrea Ferrari and Chris Beem we investigate deformations of SQED by a flat connection. In particular we study the boundary VOA, its associated variety and its module category.
- Section 4.2: in this project I construct sheaves of VOAs on BFN Coulomb branches using a presentation given in [BF23b]. The global sections of these sheaves should correspond to 3d boundary VOAs and I conjecture they satisfy a duality property between their Zhu C_2-algebra and derived endomorphism algebra.

In Section 5 I outline some future projects I am interested in investigating:

- Section 5.1: a method for computing quantum cohomology using the chiral de Rham complex.
- Section 5.2: investigating Koszul duality for chiral algebras and applications to the twisted holography program.

2 Background

2.1 $3d$ mirror symmetry

$3d$ mirror symmetry is most often studied as a duality between symplectic varieties known as the *Higgs branch* and *Coulomb branch*. Physically, these spaces are branches of the moduli space of vacua for $3d$ topological field theories (TQFTs) and contain information about the algebras of operators for these theories. These varieties are typically singular and are equipped with resolutions whose theory generalizes that of the Springer resolution $T^*(G/B) \to \mathcal{N}$. It is for this reason that Okounkov said "*Symplectic resolutions are the Lie algebras of the 21st Century*". Some notable examples of dual pairs is provided in Figure 1:

Symplectic resolution	**Symplectic dual**
$T^*\mathbb{P}^n$	$\widetilde{\mathbb{C}^2/\mathbb{Z}_{n+1}}$
$T^*(G/B)$	$T^*(G^\vee/B^\vee)$
hypertoric varieties	Gale dual hypertoric varieties
ADE quiver varieties	affine Grassmanian slices

Figure 1: Some examples of symplectic dual pairs. G is an algebraic group and $G^\vee$ denotes the Langlands dual group of G.

Symplectic duality is conjectured to interchange different algebraic and geometric data; for example one can associate a category $\mathcal{O}$ to a symplectic singularity and the duality interchanges this with the Koszul dual category $\mathcal{O}^!$ [Web19b]. For a more comprehensive overview of the subject, see the survey papers [Kam22; WY23].

2.2 Physical context, vertex algebras and open conjectures

A promising way to explore $3d$ mirror symmetry is through a conjectured relationship between vertex operator algebras (VOAs) constructed from the boundary data of the twisted theories and their Higgs and Coulomb branches [CCG18; CG19]. The boundary VOA arises by fixing holomorphic boundary conditions that support local operators forming a VOA, analogous to the correspondence between Chern-Simons theories and boundary conformal field theories.

For the A-twist of a theory with gauge group G, the boundary VOA is given by a gauged $\beta\gamma$-system VOA. One should think of the $\beta\gamma$ VOA as being the chiral[1] analogue of the Weyl algebra of differential operators and the bc VOA as being the chiral analogue of the exterior algebra of differential forms. For this reason a sheaf of $\beta\gamma$ VOAs on a space X is known as the *chiral differential operators* (CDOs) of X and a sheaf of $\beta\gamma \otimes bc$ VOAs on X is known as the the *chiral de Rham complex* (CDRs) of X.

The A-twist boundary VOA can be obtained as the global sections of the CDOs or CDRs for a quotient stack W/G, where we take W to be a G-representation. The BRST cohomology of $\beta\gamma^N \otimes bc^N$ with respect to $\mathfrak{g} = \mathrm{Lie}(G)$ corresponds to taking the global sections of a sheaf of CDRs on the symplectic reduction $T^*W /\!\!/ G$. It was posited in [CCG18] that a non-semisimple tensor category of modules for the boundary VOA corresponds to D-modules on the loop space $\mathcal{L}(T^*W/G)$ which describes the line defects of the A-twist. Local operators of the A-twist describe the functions on the Coulomb branch and these can be recovered as endomorphisms of the trivial defect.[BFN19]. This reasoning leads to the following conjecture:

Conjecture 1 ([CCG18])**.** *Let V be the boundary VOA for the A-twist of a $3d$ $\mathcal{N} = 4$ gauge theory and let $\mathcal{C}$ denote a particular non-semisimple subcategory of $V - \mathrm{mod}$. Then the Coulomb branch algebra is isomorphic to* $\mathrm{Ext}_{\mathcal{C}}(V, V)$.

In $4d$ $\mathcal{N} = 2$ superconformal QFTs there is a construction of a characteristic VOA whose associated variety recovers the Higgs branch of the theory. One can compactify such a theory to obtain a 3d $\mathcal{N} = 4$ theory with the same Higgs branch. This leads to a $3d$ mirror conjecture:

Conjecture 2. *[BF23a] Let V be the boundary VOA for the A-twist of a gauge theory with purely free fermionic boundary degrees of freedom. Then the Higgs branch is isomophic to X_V.*

Proving these conjectures is an attractive prospect as it provides a systematic method for constructing both sides of the duality from a single algebraic object. In recent work [Bal+23] Conjecture 1 was proved in the case that G is abelian and the mirror symmetry was verified at the level of module categories. However many technical difficulties remain in proving this statement for the non-abelian case. In Section 3.2 I provide a summary of recent work with Andrea Ferrari where we proved Conjecture 2 for the case of super quantum electrodynamics (SQED). In Section 1 I discuss ongoing work towards understanding the non-abelian cases for both conjectures.

3 Recent work

3.1 Tilting generator for the $T^*\mathrm{Gr}(2, 4)$ Coulomb branch

As suggested in Section 2.2, the (derived) category of line defects for the twisted theory is described by the (derived) category of coherent sheaves on the Coulomb branch. In [SW24], my advisor and I studied this category for the quiver gauge theory associated to the quiver seen in Figure 2 which has (resolved) Coulomb branch $T^*\mathrm{Gr}(2, 4)$. Our goal was to provide a concrete algebraic description of the derived category of coherent sheaves $D^b_{\mathrm{coh}}(T^*\mathrm{Gr}(2, 4))$.

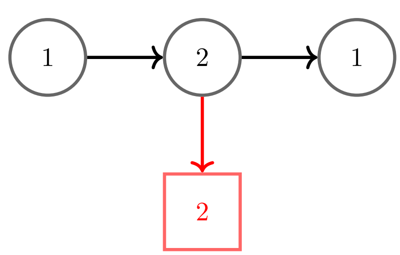

Figure 2: The framed quiver determining the gauge and matter content for the quiver gauge theory with resolved Coulomb branch $T^*\mathrm{Gr}(2, 4)$.

[1] We use adjective "*chiral*" here to mean the vertex algebra analogue of that object.

We achieved this using techniques for studying non-commutative resolutions of symplectic varieties over fields of characteristic $p > 0$ developed in [BK07]. Given a non-commutative resolution R of a symplectic variety X, it was shown in [Kal06] that there exists a coherent sheaf $\mathcal{E}$ called a *tilting generator* such that $R = \mathrm{End}(\mathcal{E})$ and provides an equivalence of derived categories

$$\begin{aligned} D^b_{\mathrm{coh}}(X) &\to D^b(R\text{--}\mathrm{mod}^{\mathrm{fg}}) \\ \mathcal{F} &\mapsto \mathrm{RHom}(\mathcal{E}, \mathcal{F}). \end{aligned}$$

In [Web19a] these methods were used to show that any BFN Coulomb branch $\mathfrak{M}$ has a non-commutative resolution A with tilting generator $\mathcal{Z}$ such that for any BFN resolution $\tilde{\mathfrak{M}} \to \mathfrak{M}$ there is a derived equivalence[2]

$$D^b_{\mathrm{coh}}(\tilde{\mathfrak{M}}) \cong D^b(A\text{--}\mathrm{mod}^{\mathrm{fg}}). \tag{1}$$

In the sequel [Web22] it was shown that if the Coulomb branch is that of a quiver gauge theory, then A is a *cylindrical KLRW algebra*, a type of diagrammatic algebra generated by string diagrams on a cylinder.

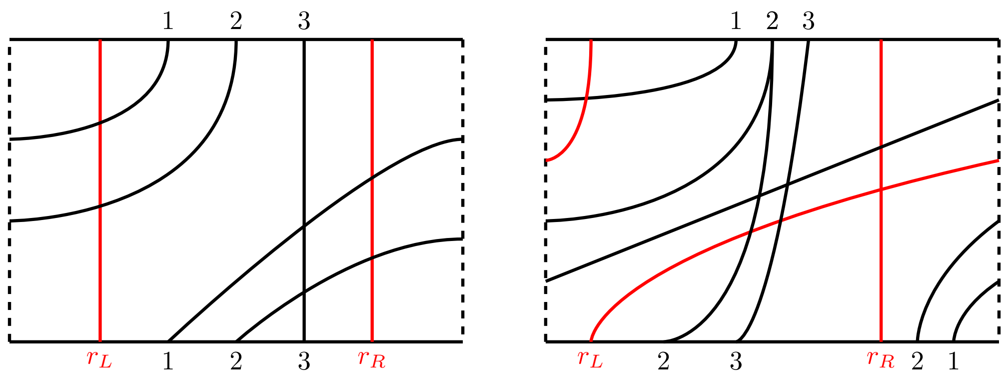

Figure 3: Examples of cylindrical KLRW diagrams. The left diagram is an element of the algebra A while the right diagram is an element of an A-module labelled by the idempotent 223221. The colours and labels indicate the quiver data that a strand corresponds to and action of the algebra is given by stacking diagrams from the top.

The aforementioned constructions however are not explicit and so a major motivation of this project was to provide a concrete construction of the tilting generator $\mathcal{Z}$ for $D^b_{\mathrm{coh}}(T^*\mathrm{Gr}(2,4))$. This was achieved by exploiting the geometry of $\mathrm{Gr}(2,4)$, and constructing six modules for the KLRW algebra A corresponding to idempotents labelled by strings of quiver vertices. These modules were identified with coherent sheaves on $\mathrm{Gr}(2,4)$ as summarised in Table 4.

Coulomb branch sheaf	**KLRW idempotent**	**Grassmanian sheaf**
$\mathcal{Z}_1$	$212^{(2)}32$	$\mathcal{O}$
$\mathcal{Z}_2$	$2212^{(2)}3$	$\mathcal{L}^{-1}$
$\mathcal{Z}_3$	221322	$\mathcal{H} \otimes \mathcal{L}$
$\mathcal{Z}_4$	222312	$\mathcal{H}$
$\mathcal{Z}_5$	223221	$\mathcal{T}^{\perp}$
$\mathcal{Z}_6$	221223	$\mathcal{T}$

Figure 4: Table showing the coherent sheaves on $T^*\mathrm{Gr}(2,4)$ corresponding to KRLW algebra modules. The idempotent labels indicate the order of the strands appearing in the KLRW diagrams for that module. $\mathcal{L}$ is the unique ample line bundle generating the Picard group of $T^*\mathrm{Gr}(2,4)$, $\mathcal{T}$ denotes the tautological bundle on $\mathrm{Gr}(2,4)$ and $\mathcal{L}^{-2} \to \mathcal{H} \to \mathcal{O}$ is a unique non-trivial extension of vector bundles.

It is worth noting that this collection of coherent sheaves is different to those constructed in [Kap83] by applying Schur functors to $\mathcal{T}$. Utilising the characteristic p quantization methods developed in [Kal06; BK07] and applied to the Coulomb branch setting in [Web19a], we argue the following result:

[2] While the proof of this statement makes use of quantization in characteristic p, the result holds in characteristic 0.

Theorem 1 ([SW24]). *The coherent sheaf on $T^*\mathrm{Gr}(2,4)$ given by*

$$\mathcal{Z} = \mathcal{Z}_1 \oplus \cdots \oplus \mathcal{Z}_6$$

is a tilting generator that realises the derived equivalence (1).

3.2 Associated variety of $V_1(\mathfrak{psl}(N|N))$ & 3d SQED Higgs branch

In [FS24], we verified Conjecture 2 for the case of SQED with $N > 2$ hypermultiplets. To accomplish this we proved the following two theorems:

Theorem 2 ([FS24]). *The boundary VOA for the A-twist of 3d SQED is*

$$V_A = H^{\frac{\infty}{2}+0}(\mathfrak{gl}_1, \widetilde{\mathfrak{gl}_1}, \beta\gamma^N \otimes bc^N) \cong L_1(\mathfrak{psl}(N|N)).$$

Theorem 3 ([FS24]).

$$X_{L_1(\mathfrak{psl}(N|N)} \cong \overline{\mathbb{O}_{\min}(\mathfrak{sl}(N))}.$$

The first theorem involved identifying the BRST reduction with the simple quotient of $V^1(\mathfrak{psl}(N|N))$. This was achieved using free field realizations of the BRST cohomology provided in [Bal+23] and results expressing $L_1(\mathfrak{psl}(N|N))$ as a simple current extension of $L_1(\mathfrak{sl}(N)) \otimes L_{-1}(\mathfrak{sl}(N))$ [AM18; Ada+19; CY21].

The second theorem identifies the associated variety of the boundary VOA with the closure of the minimal nilpotent orbit of $\mathfrak{sl}(N)$. We accomplished this by identifying a singular vector in $V^1(\mathfrak{psl}(N|N))$ generating the maximal submodule of the even subalgebra $V^1(\mathfrak{sl}(N)) \otimes V^{-1}(\mathfrak{sl}(N))$.[3] The associated variety of this subalgebra was shown to be the closure of the sheet containing the minimal nilpotent orbit as a codimension 1 subvariety [AM19b]. We demonstrated that the new singular vector also generated elements in the Zhu C_2-algebra corresponding to the functions vanishing on $\overline{\mathbb{O}_{\min}(\mathfrak{sl}(N))}$ but not the sheet.

4 Current work

4.1 Deformed boundary VOAs

In collaboration with Andrea Ferrari and Christopher Beem, we are investigating deformations of the A-twist for 3d SQED by a flat connection. Considering deformations of the BRST differential by a scalar leads us to conjecture the following:

Conjecture 3. *Let $\lambda \in \mathbb{C}$ be the deformation parameter for the BRST differential, μ be the associated moment map and denote by V^A_λ the boundary VOA for the A-twist of 3d SQED. Then*

$$X_{V^A_\lambda} \cong \mu^{-1}(\lambda) /\!\!/ \mathbb{C}^\times$$

which is the Higgs branch of the deformed theory.

When $\lambda = 0$, we recover the result of [FS24]. Another aspect of this work is to understand the category of V^A_λ-modules corresponding to line operators of the deformed theory. The structure of this module category is simpler in the deformed case and this should be reflected in the Coulomb branch algebra. Preliminary results lead us to conjecture:

Conjecture 4. *For an abelian theory with $\lambda \neq 0$, the category of V^A_λ-modules $\mathcal{C}^A_\lambda$ is semisimple.*

The proof of this statement appears to follow from the construction given in [Bal+23] for $\mathcal{C}^A$ as an induction (or gauging) of the $\beta\gamma$-module category $\mathcal{C}_{\beta\gamma}$. The ungauged category has a block decomposition with respect to generalized eigenvalues of the current operator:

$$\mathcal{C}_{\beta\gamma} = \bigoplus_{\lambda \in \mathbb{C}/\mathbb{Z}} \mathcal{C}_{\beta\gamma,\lambda}.$$

The gauged cateogry $\mathcal{C}^A$ is the image of the subcategory of monodromy free objects $\mathcal{C}_{\beta\gamma}$ under an induction functor. In the undeformed case this source subcategory lies within $\mathcal{C}_{\beta,\gamma,0}$ which contains so-called "atypical" modules which have interesting non-trivial extensions. In the deformed case the source category is a "typical" block whose

[3] I conjecture that this submodule is maximal and am looking to prove this and other generalizations in future work.

non-trivial extensions are removed by the monodromy condition.

One motivation for studying these deformations relates to the mirror symmetry between the derived endomorphism algebra and the Zhu C_2-algebra. On one hand, the C_2-algebra of V^A describes the Higgs branch of the theory and provides information regarding finiteness properties for V^A. On the other hand, the derived endomorphism algebra of V^A describes the Coulomb branch and provides information regarding extensions of V^A. We hope that investigating deformations where one of these structures becomes particularly simple will elucidate the relationship between finiteness conditions and extensions for VOAs.

4.2 Boundary VOAs via Chiral quantization of Coulomb branches

A VOA V is said to be a *chiral quantization* of a variety X if its associated variety X_V is isomorphic to X [AKM15; AM19a]. This definition raises the following questions:

Question 1. *Given a symplectic variety X, when does there exist a VOA V such that $X_V = X$? How many non-isomorphic chiral quantizations does a given X have?*

Restricted versions of these questions have been answered in the cases where V is given by taking CDOs or the CDR complex of Y where $X = T^*Y$. In the case of CDOs the second Chern character of Y is an obstruction to the existence of CDOs on Y [GMS99]. When it exists, CDOs can be constructed analogously to Fedosov quantization [GGW20]. For the CDR complex, no such obstruction is present.

A series of papers [AKM15; Kuw17; AKM23] constructs CDOs and CDRs on cotangent bundles of flag varieties, intersections of Slodowy slices with the nilcone, hypertoric varieties and Hiblert schemes. The authors accomplish these “microlocal chiral quantizations” using sheaves of filtered $\hbar$-adic VOAs and exploiting the Poisson geometry of the associated varieties and their arc spaces.

In another recent paper [BF23b] it was proved that any BFN Coulomb branch is isomorphic to the equivariant Hilbert scheme of a hypertoric variety. Using the chiral quantizations of hypertoric varieties and Hilbert schemes given in [Kuw17; AKM23] I am working towards proving the following:

Conjecture 5. *Given a BFN Coulomb branch $\mathfrak{M}_C$, there exists a sheaf of $\hbar$-adic VOAs $\mathcal{V}$ on $\mathfrak{M}_C$ such that the global sections $V = \Gamma(\mathcal{V}, \mathfrak{M}_C)$ give a chiralization of $\mathfrak{M}_C$.*

Here it is expected that V will be the B-side boundary VOA and a mirror construction could be performed using the Higgs branch $\mathfrak{M}_H = \mathfrak{M}_C^!$ to construct the A-side boundary VOA. This leads one to conjecture:

Conjecture 6. *Letting $V^! = \Gamma(\mathcal{V}, \mathfrak{M}_H)$ and denoting by R_V the Zhu C_2-algebra of V, there are algebra isomorphisms*

$$\mathrm{Ext}_{\mathcal{C}^A}(V, V) \cong R_{V^!}, \qquad R_V \cong \mathrm{Ext}_{\mathcal{C}^B}(V^!, V^!).$$

Such a result could provide deep insights into the structure of VOAs appearing in this manner. Once Conjecture 5 is proved, the construction could also be used to prove Conjectures 1 and 2 as well as to explore the answer to Question 1. To construct the V we exploit the distinguished open cover used in [Kuw17] to construct chiral quantizations of hypertoric varieties and transport these sheaves of $\hbar$-adic VOAs to the equivariant Hilbert scheme of the hypertoric variety.

Another interesting question to consider here is how the derived structure of the Higgs branch is encoded in the boundary VOA. I am looking to understand this by computing the Zhu C_2-algebra at the level of BRST cochains and checking the resulting dg structure is correct.

5 Future work

In this section, I will outline some ideas for future research making use of the mathematical tools used in my prior work.

5.1 Quantum cohomology from the chiral de Rham complex

In [FL06] the authors explore ($2d$) mirror symmetry for toric varieties using an intermediate “*I-model*” in addition to the usual A and B-models. Their method may be roughly surmised as showing the equivalence[4] of the A and

[4] The equivalence between the A and I-models is as conformal field theories (that is the correlation functions of both theories agree), but the equivalence between the I and B-model is more subtle.

B-models to the I-model, rather than demonstrating mirror symmetry between the A and B-models directly. While the physical arguments made are quite general, the utility of the I-model has not been widely explored mathematically outside of the case of toric varieties.

It was shown [FL06] that the I-model on a toric variety X comes equipped with a family of q-deformed differentials $d(q)$[5]. For $q \neq 0$ it was shown that the $d(q)$-cohomology is the quantum cohomology of X, while the $d(0)$-cohomology produces the CDR complex of X [MS00]. The goal of this project would be to investigate the following:

Question 2. *For what symplectic varieties X can the quantum cohomology of X be expressed as the the q-deformed comhology of the CDR complex of X?*

The algebra of operators for the theories in studied in [FL06] are expressed using an algebraic structure that can be thought of as a VOA with "mixed" chiral and anti-chiral parts. Such objects have not to my knowledge been studied widely outside of a few cases [KO03] and it is possible that re-framing these structure in the language of factorization algebras of observable as described in [CG16] would be a fruitful approach to generalizing this story for more general symplectic varieties.

5.2 Chiral Koszul duality and twisted holography

Koszul duality is a well-known duality between algebras as well as their module categories, most famously known for exchanging symmetric and exterior algebras and exchanging universal enveloping algebras of Lie algebras with their Chevalley-Eilenberg complex. In more recent years, Koszul duality has found applications in QFT, particularly in the study of defects.

An overview of how Koszul algebras can describe defects in quantum theories is given in [PW23] and I will briefly outline the explanation presented there. Consider a system with an algebra of local operators A and product given by the OPE along a line L[6]. Suppose you also have a quantum mechanical system on L with algebra of observables B. The tensor product $A \otimes B$ can be interpreted as the uncoupled algebra of the combined system on L, while a coupling is a choice of map

$$\varphi : A^! \to B$$

where $A^!$ denotes the Koszul dual of A. This statement is equivalent to saying that that φ satisfies the *Maurer-Cartan equation*. Physically we can interpret this to mean that the Koszul dual $A^!$ is the algebra of the universal line defect on L and a choice of coupling φ determines a twisted tensor product $A \otimes_\varphi B$ describing the coupled system.

Physically, we are interested in generalizing this story. For example, now suppose that L is a complex line and the theory is holomorphic on L. The algebra of operators will now form a VOA V and we expect the universal defect on L to be given by a *chiral Koszul dual* $V^!$. Unfortunately Koszul duals have not been defined in general for VOAs. Mathematically, one can see why this is in two ways. The first is that in the case of associative algebras, Koszul duals can be defined by taking qutoients of a free algebra, in this case the tensor algebra. However a notion of free VOA does not exist since the locality axiom cannot be expressed as a finite set of identities [Roi01]. Another way to view the issue is that is VOAs are not algebras over an operad, rather they are modules over a partial operad [HL93] and so Koszul duality for operads also fails to solve the problem.

Despite this, physical expectations show us that computing the Maurer-Cartan elements of certain VOAs produces the correct structures and in [GLZ22] a definition of Koszul duality for chiral algebras defined by quadratic data is given. The goal for this project is to construct new examples by looking at deformations and BRST reductions of known examples. Doing this will provide information about the general structure and properties of chiral Koszul duality, as well as providing new examples for use in physics. In particular, chiral Koszul duality is expected to play a role in the twisted and celestial holography programs [CP21; CP22].

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